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1992 J. Phys. A: Math. Gen. 25 2261

(http://iopscience.iop.org/0305-4470/25/8/036)

J. Phys. A: Math. Gen. 25 (1992) 2261–2272. Printed in the UK

Generalized deformed oscillator corresponding to the modified Pöschl–Teller energy spectrum

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Received 16 July 1991

Abstract. A generalized deformed oscillator, with eigenvalues equal to the eigenvalues of the Schrödinger equation with the modified Pöschl–Teller potential, is constructed. For special values of the potential depth the deformed oscillator algebra has a finite-dimensional irreducible representation. The polynomial representation and the associate deformed operations of integration and differentiation are studied. The results of this study are general and they can be applied directly to the case of the q-deformed oscillator, with q being a root of unity.

1. Introduction

The quantum algebra $SU_q(2)$ was introduced by Kulish and Reshetikhin (1983). Biedenharn (1989) introduced the q-deformed harmonic oscillator and constructed a realization of $SU_q(2)$. This was also done independently by Macfarlane (1989). The q-deformed harmonic oscillator was initially considered to be an intermediate step in the study of deformed quantum algebras such as $SU_q(2)$, $U_q(1,1)$, etc whose applications are relevant in inverse problems and other branches of physics.

Recently attention has been focused on quantum mechanical systems for which the properties can be described by the q-deformed oscillator. Floratos and Tomaras (1990) have shown, that, if a particle moves in the field of a shielded magnetic flux on a discetized cycle, then its Hamiltonian corresponds to the energy spectrum of a q-deformed oscillator. Bonatsos *et al* (1991b) have studied the hydrogen molecular spectrum, using the q-deformed anharmonic oscillator energy spectrum. Bonatsos *et al* (1991a) found that the potential with the same WKB spectrum as the q-deformed oscillator has similarities with the modified Pöschl–Teller potential. The Pöschl–Teller potential is a widely used potential in many branches of physics (see references reported by Frank and Wolf (1985)).

In a recent paper (Daskaloyannis 1991), it was shown that an energy spectrum corresponds to a generalized deformed oscillator algebra. This algebraic structure can be constructed explicitly. The problem, which normally arises, is the construction of generalized deformed oscillators corresponding to well known potentials and the study of the correspondence between the properties of the conventional potential picture and the algebraic one using deformed oscillators. The correspondence between the algebraic and the potential pictures for an arbitrary energy spectrum is analogous to the correspondence between the potential model and the creation and destruction operator algebra of the harmonic oscillator.

0305-4470/92/082261+12$04.50

In this paper, starting from an arbitrary deformation of the oscillator, I construct the appropriate deformed oscillator algebra corresponding to the modified Pöschl–Teller potential. In section 2, for clarity reasons, I present a short description of the generalized deformed oscillator algebra which is studied in a previous paper (Daskaloyannis 1991). In section 3 I construct the deformed oscillator algebra which has an energy spectrum equivalent to the Pöschl–Teller spectrum. For special values of the potential depth finite-dimensional irreducible representations of the deformed algebra exist and this case is analogous to the case of the q-deformed oscillator with q being a root of unity. In section 4 the polynomial basis and the deformed analysis are defined for the deformed algebra, a finite-dimensional irreducible representation of this algebra exists for special values of the potential strength. The results of this section can be easily applied to every finite-dimensional irreducible representation of the ordinary q-deformed oscillator with q being a root of unity. This topic has not been studied extensively; a recent approach to this subject can be found in Baulieu and Floratos (1991) in which they use the notion of a quantum plane. In the present paper this notion is not used but instead the work is done in the space of the truncated entire functions.

2. The generalized deformed oscillator algebra

A general deformation of the harmonic oscillator can be given by the basic relation:

$$f(aa^\dagger) - f(a^\dagger a) = 1 \tag{1}$$

where a and $a^\dagger$ are conjugate operators, $f(x)$ is a real analytic function defined on the real positive axis. In the ordinary oscillator algebra the function $f(x)$ is defined by

$$f(x) = x \tag{2}$$

which leads to the commutation relation:

$$[a, a^\dagger] = 1$$

The *number* operator N, by definition, satisfies the commutation relations

$$[a, N] = a \qquad \text{and} \qquad [a^\dagger, N] = -a^\dagger. \tag{3}$$

It can be shown that this operator is given by the relation

$$N = f(a^\dagger a). \tag{4}$$

If equation (1) is true then the following relation is also true

$$aa^\dagger = g(a^\dagger a) \tag{5}$$

where the function $g(x)$ is defined by

$$g(x) = F(1 + f(x)) \qquad \text{and} \qquad F = f^{-1}. \tag{6}$$

By induction the following relations can be proved:

$$[a,(a^\dagger a)^n] = ((g(a^\dagger a))^n - (a^\dagger a)^n)a$$

and

$$[a^\dagger,(a^\dagger a)^n] = -a^\dagger((g(a^\dagger a))^n - (a^\dagger a)^n).$$

These equations imply that

$$[a, f(a^\dagger a)] = (f(g(a^\dagger a)) - f(a^\dagger a))a \tag{7a}$$

and

$$[a^\dagger, f(a^\dagger a)] = -a^\dagger(f(g(a^\dagger a)) - f(a^\dagger a)). \tag{7b}$$

Thus the *number* operator $N = f(a^\dagger a)$ satisfy equations (3).

Let us assume that $|\alpha\rangle$ is a base of eigenvectors of the number operator N

$$N|\alpha\rangle = \alpha|\alpha\rangle \tag{8}$$

then equations (3) imply that the operator a (or $a^\dagger$) is a destruction (or a creation) operator such that

$$a|\alpha\rangle = \sqrt{[\alpha]}|\alpha - 1\rangle \qquad a^\dagger|\alpha\rangle = \sqrt{[\alpha+1]}|\alpha+1\rangle \tag{9}$$

where $[\alpha]$ is a function of α. Furthermore from equation (6) we can find that

$$[\alpha+1] = g([\alpha]) \qquad \text{or} \qquad f([\alpha+1]) = 1 + f([\alpha]). \tag{10}$$

Thus finally from these equations, we conclude that

$$[\alpha] = F(\alpha). \tag{11}$$

The eigenvector $|0\rangle$, corresponding to the zero eigenvalue of the number operator N, satisfies the following relation:

$$\text{if } F(0) = 0 \text{ (or } f(0) = 0\text{) then } a|0\rangle = 0. \tag{12}$$

In this paper we assume that the function $F(x)$ is zero for $x = 0$.

The eigenvectors of the number operator $N = f(a^\dagger a)$ are generated by the formula

$$|n\rangle = \frac{1}{\sqrt{[n]!}}(a^\dagger)^n|0\rangle \tag{13}$$

where

$$[n]! = \prod_{k=1}^{n}[k] = \prod_{k=1}^{n} F(k).$$

These eigenvectors are also eigenvectors of the *energy* operator:

$$H = (A/2)(a^\dagger a + a a^\dagger) \tag{14}$$

corresponding to the eigenvalues

$$E_n = (A/2)([n+1] + [n]) = (A/2)(F(n+1) + F(n)). \tag{15}$$

Let us consider that the energy spectrum is given by a definite real function of the number $n + \frac{1}{2}$:

$$E_n = (A/2)H(n + \tfrac{1}{2})$$

then

$$H(x + \tfrac{1}{2}) = (F(x+1) + F(x))/2. \tag{16}$$

The solution of this equation is a difficult task in the general case. Many years ago Buck (1946) (quoted in the classical book of Boas and Buck (1964)) gave a theorem for the existence of solutions for the simple difference equation

$$y(z+1) - \beta y(z) = h(z). \tag{17}$$

Unfortunately Buck's existence theorem cannot be applied in our case, because the number β was assumed to be a complex number not belonging in the negative real axis, while in our case $\beta = -1$, see equation (16). We can implicitly construct the solution of equation (16) in many cases corresponding to energy spectra with physical significance, which holds for the case of the simple or the q-deformed (with real q) oscillator, or for the ordinary harmonic oscillator. Apart from this reference to the difference equation (16), we do not know any other studies on this type of simple functional equation. Special cases of equation (17) have been studied by different authors. Cutright and Zachos (1990) solved the equation

$$r^2 f(x) - f(x+1) = r$$

and after finding the solution $f(x)$ they solved

$$H(x) - s^2 H(x+1) = s f(x).$$

We must point out that theoretically a solution of equation (16) exists in many cases. Assuming that

$$F(0) = 0$$

we can find the values of the function $F(x)$ if $x = n$ is a natural number:

$$F(n) = 2\sum_{k=1}^{n} (-1)^{n+k} H\left(\frac{2k-1}{2}\right).$$

A similar method was initially used by Jannussis (1991) for the solution of similar equations. We must point out that the function $F(x)$ will act on the number operator N and for consistency reasons this function should be defined over all the complex plane and not only on the set of natural numbers $n = 0, 1, \ldots$. Using standard techniques of complex analysis (see Ahlforls (1975)), an entire function $P(z)$ can be found such that $P(n) = F(n)$ for n a natural number.

Here we shall study the case, which is hierarchically more complicated than the simple harmonic oscillator, i.e. the case where the function $H(z)$ is assumed to be a second degree polynomial. This more complicated spectrum has a physical counterpart, because the well known Pöschl–Teller and the Morse potentials correspond to a quadratic function $H(z)$.

In the literature one can find other general treatments of deformed oscillators which are compatible with the method proposed here. Kuryshkin (1980), quoted by Jannussis *et al* (1982a), has studied the case where

$$aa^\dagger = \sum_{k=1}^{m} Q_k a^{\dagger^k} a^k \qquad \text{with } Q_k \text{ real, } k = 1, 2, \ldots, m.$$

Jannussis (1991) studied the case where the commutation relation is given by

$$AA^\dagger - QA^\dagger A = f(n_b) \qquad \text{where } n_b = b^\dagger b,\ [b^\dagger, b] = 1.$$

Jannussis *et al* (1982b) have studied the case

$$A^\dagger = b^\dagger f(n_b) \qquad A = f(n_b)b.$$

All these deformation schemes give compatible results.

3. The deformed oscillator equivalent to the Pöschl–Teller spectrum

In the case of the modified Pöschl–Teller potential

$$V(x) = D \tanh^2(x/R)$$

the energy spectrum is given by

$$E_n = \frac{\hbar^2}{mR^2}\left\{-\frac{1}{8} + \frac{n+\frac{1}{2}}{2}\sqrt{\frac{8mDR^2}{\hbar^2}+1} - \frac{(n+\frac{1}{2})^2}{2}\right\}$$

see ter Haar (1975). Then we can fix

$$\begin{aligned} A &= \frac{\hbar^2}{2mR^2}\left(\sqrt{8mDR^2/\hbar^2+1} - 1\right) \\ H(x) &= \left(-\tfrac{1}{4} + x\sqrt{8mDR^2/\hbar^2+1} - x^2\right)/\left(\sqrt{8mDR^2/\hbar^2+1} - 1\right) \end{aligned} \tag{18}$$

while the function $F(x)$, which is the solution of equation (16), exists and is given by

$$F(x) = x(\sqrt{8mDR^2/\hbar^2+1} - x)/(\sqrt{8mDR^2/\hbar^2+1} - 1). \tag{19}$$

It should be pointed out that, for special values of the potential depth D, the deformed oscillator accepts a finite irreducible boson representation. Let us suppose that a natural number p exists such that

$$8mDR^2/\hbar^2 + 1 = p^2 \qquad \text{or} \qquad D = (p^2-1)\hbar^2/8mR^2 \tag{20}$$

then

$$F(p) = [p] = 0 \tag{21}$$

and the deformed oscillator accepts only a definite number of eigenvectors:

$$|0\rangle, |1\rangle, |2\rangle, \ldots, |p-1\rangle$$

while the algebra of the operators $\{a, a^\dagger, N, 1\}$ is a cyclic one for the operators a and $a^\dagger$:

$$(a^\dagger)^k = 0 \qquad (a)^k = 0 \qquad \text{if } k \geqslant p.$$

Analogous situations are found in the case of the q-deformed oscillator, where irreducible boson representations exist when $q = \exp(\mathrm{i}2\pi/p)$ where p is a natural number (see Arnaudon and Chakrabati (1991), Yan Hong (1990) and Floratos and Tomaras (1990)).

Using this basis $\{|n\rangle\}$, $n = 0, 1, \ldots, p-1$ we can construct a matrix representation of the operators a and $a^\dagger$, N and H:

$$\begin{aligned} \langle n|a^\dagger|m\rangle &= \delta_{n,m+1}\sqrt{n(p-n)/(p-1)} \\ \langle n|H|m\rangle &= \delta_{n,m}\frac{\hbar^2}{2mR^2}[2p(n+\tfrac{1}{2}) - (n+\tfrac{1}{2})^2 - \tfrac{1}{4}] \\ \langle n|N|m\rangle &= \delta_{n,m}n. \end{aligned} \tag{22}$$

4. Polynomial basis for the generalized deformed oscillator

In this section we shall discuss the polynomial basis of the finite deformed oscillator equivalent to the Pöschl–Teller potential, obeying restriction (21). The results of this section can be directly generalized to the case of the ordinary q-deformed oscillator with q being a root of unit. This topic is still being investigated (see Hong Yan (1990) and Jian-Hui *et al* (1991)), while the case with q a real number has been widely studied by several authors (Bracken *et al* 1991, Sun and Ge 1991, Hong Yan 1990). Recently Baulieu and Floratos (1991) have studied this topic using the notion of a quantum plane, and they have introduced a differential and an integral for the holomorphic representations of the deformed oscillator. In the general deformed case the quantum plane is not defined. Our proposed method does not imply the notion of a quantum plane.

Let $\mathcal{H}$ be the set of the entire functions

$$f(z) = \sum_{n=0}^{\infty} a_n z^n.$$

The projection operator J_k projects the function $f(z)$ onto the truncated polynomial $J_k f(z)$ of degree k:

$$J_k f(z) = \sum_{n=0}^{k} a_n z^n \in J_k\mathcal{H}.$$

The space spanned by the deformed oscillator basis $|n\rangle$ is equivalent to the space $J_{p-1}\mathcal{H}$ spanned by the basis:

$$\frac{z^n}{\sqrt{[n]!}} \qquad n = 0, 1, \ldots, p-1. \tag{23}$$

For simplicity, we shall omit the projection symbol J_{p-1}, keeping in mind that we shall refer to polynomials inside the subspace $J_{p-1}\mathcal{H}$.

Any function $f(z) \in J_{p-1}\mathcal{H}$ can be written as

$$f(z) = \sum_{k=0}^{p-1} f_n \frac{z^n}{[n]!} \equiv \langle z|f\rangle \tag{24}$$

where

$$\langle z| \equiv \sum_{k=0}^{p-1} \frac{z^n}{\sqrt{[n]!}} \langle n| \tag{25}$$

and

$$|f\rangle \equiv \sum_{k=0}^{p-1} \frac{f_n}{\sqrt{[n]!}} |n\rangle. \tag{26}$$

The element $|z\rangle$ is the *coherent* (but not normalized) eigenstate of the destruction operator a with eigenvalue $\bar{z}$:

$$a|z\rangle = \bar{z}|z\rangle. \tag{27}$$

The multiplication of the function $f(z)$ by z can be regarded as an application from the space of entire functions $\mathcal{H}$ into $\mathcal{H}$. The restriction of this application in the space $J_{p-1}\mathcal{H}$ is formally represented by $J_{p-1} z J_{p-1}$ and this operation corresponds to

$$z \sum_{n=0}^{p-1} a_n z^n = \sum_{n=1}^{p-1} a_{n-1} z^n \in J_{p-1}\mathcal{H} - J_0\mathcal{H}. \tag{28}$$

The derivative $\partial/\partial z$ is also an application defined in the space $\mathcal{H}$ and its restriction in $J_{p-1}\mathcal{H}$ is easily calculated:

$$\left(\frac{\partial}{\partial z}\right)\sum_{n=0}^{p-1} a_n z^n = \sum_{n=0}^{p-2}(n+1)a_{n+1}z^n \in J_{p-2}\mathcal{H}. \tag{29}$$

In the space $J_{p-1}\mathcal{H}$ we can define the operator

$$\frac{\partial}{\partial_{\mathrm{D}} z} \boxminus \frac{1}{z}F\left(z\circ\frac{\partial}{\partial z}\right) \tag{30}$$

and then without difficulty we can show that

$$\frac{\partial}{\partial_{\mathrm{D}} z}z^n = F(n)z^{n-1} \qquad \text{if } 0 < n \leqslant p-1$$
$$\frac{\partial}{\partial_{\mathrm{D}} z}z^0 = 0.$$

Expansion (24) of the function $f(z)$ corresponds to the deformed Taylor expansion around zero because we can easily show that

$$f_n = \left[\left(\frac{\partial}{\partial_{\mathrm{D}} u}\right)^n f(u)\right]_{u=0} \equiv f^{[n]}(0). \tag{31}$$

The operator $z\circ\partial/\partial_{\mathrm{D}} z$ defined by equation (30) is a one-to-one application in the subspace $J_{p-1}\mathcal{H} - J_0\mathcal{H}$; therefore the inverse of this operator exists in this subspace and it is given by

$$\left(z\circ\frac{\partial}{\partial_{\mathrm{D}} z}\right)^{-1} z^n \equiv \frac{1}{F(n)}z^n \qquad \text{if } 0 < n \leqslant p-1.$$

Using this operator the integration operator Int can be defined by

$$\mathrm{Int} \equiv \left(z\circ\frac{\partial}{\partial_{\mathrm{D}} z}\right)^{-1} \circ z \tag{32}$$

and without difficulty the following relation can be shown:

$$\mathrm{Int}\, z^n \equiv \frac{z^{n+1}}{F(n+1)} \qquad \text{if } 0 \leqslant n \leqslant p-2.$$

For any function $f(z)$, given by equation (24), we can define the integral

$$\mathrm{Int}\, f(z) \equiv \int_0^z f(u)\,\mathrm{d_D}u = \sum_{n=1}^{p-1} f_{n-1}\frac{z^n}{[n]!}$$

and by definition

$$\int_a^b f(u)\mathrm{d_D}u = \mathrm{Int}\, f(b) - \mathrm{Int}\, f(a). \tag{33}$$

The definition of the coherent state (25) and (27) implies that the deformed exponential function is defined by the following formula:

$$\exp_{\mathrm{D}}(z\bar{w}) \equiv \langle z|w\rangle = \sum_{n=0}^{p-1} \frac{(z\bar{w})^n}{[n]!} \tag{34}$$

and without difficulty we can show the deformed generalizations of the usual identities:

$$\int_0^z \left(\frac{\partial}{\partial_{\mathrm{D}} u}\right) f(u)\,\mathrm{d}_{\mathrm{D}} u = f(z) - f(0)$$

$$\left(\frac{\partial}{\partial_{\mathrm{D}} z}\right) \int_0^z f(u)\,\mathrm{d}_{\mathrm{D}} u = f(z) - f^{[p-1]}(0)/[p-1]!$$

$$\left(\frac{\partial}{\partial_{\mathrm{D}} w}\right)^n \exp_{\mathrm{D}}[wz] = z^n \exp_{\mathrm{D}}[wz]$$

$$\int_0^z \exp_{\mathrm{D}}[wu]\,\mathrm{d}_{\mathrm{D}} u = w^{-1}(\exp_{\mathrm{D}}[wz] - 1)$$

$$\int_0^z u^n \exp_{\mathrm{D}}[wu]\,\mathrm{d}_{\mathrm{D}} u = \left(\frac{\partial}{\partial_{\mathrm{D}} w}\right)^n [w^{-1}(\exp_{\mathrm{D}}[wz] - 1)].$$

The space $J_{p-1}\mathcal{H}$ also has the structure of a finite dimensional Hilbert space with the product:

$$\langle f|g\rangle \equiv \left[\bar{f}\left(\frac{\partial}{\partial_{\mathrm{D}} z}\right) g(z)\right]_{z=0}. \tag{35}$$

This structure is compatible with expressions (24)–(26).

This product can be formally generated by introducing a measure $\mathrm{d}\mu(\bar{z}, z)$ on the complex z plane having the following property:

$$\int \mathrm{d}\mu(\bar{z}, z)\bar{z}^n z^m \equiv \delta_{n,m}[n]! \tag{36}$$

and

$$\langle f|g\rangle = \int \mathrm{d}\mu(\bar{z}, z)\bar{f}(\bar{z})g(z). \tag{37}$$

The basic properties of this measure can be easily deduced from equation (36); we shall list here only the most fundamental ones. If A is an operator defined on the finite-dimensional Hilbert space spanned by the vectors $|n\rangle$ then there is a matrix representation defined by

$$A = \sum_{m,n=0}^{p-1} A_{n,m}|n\rangle\langle m|. \tag{38}$$

This operator corresponds to a kernel $A(w,u)$ acting on the space $J_{p-1}\mathcal{H}$:

$$A(w,\bar{u}) \equiv \langle w|A|u\rangle = \sum_{m,n=0}^{p-1} A_{n,m}\langle w|n\rangle\langle m|u\rangle$$
$$= \sum_{m,n=0}^{p-1} A_{n,m}\frac{w^n}{\sqrt{[n]!}}\frac{\bar{u}^m}{\sqrt{[m]!}} \tag{39}$$

and

$$\langle w|A|f\rangle = \int d\mu(\bar{z},z)A(w,\bar{z})f(z) \tag{40}$$

where the function $f(z)$ is defined by equations (24) and (26).

The product of two operators A and B corresponds to the *convolution* of the corresponding kernels:

$$\langle w|AB|u\rangle = \int d\mu(\bar{z},z)A(w,\bar{z})B(z,\bar{u}). \tag{41}$$

This equation implies the following *resolution* of the identity:

$$\int d\mu(\bar{z},z)|z\rangle\langle z| = \sum_{n=0}^{p-1}|n\rangle\langle n| \equiv \mathbf{1}_{p-1} \tag{42}$$

where $\mathbf{1}_{p-1}$ is the unity in the finite-dimensional Hilbert space spanned by the vector basis $|n\rangle$. The following relations can be proved without difficulty, using the definition (34) of the deformed exponential function:

$$\int d\mu(\bar{z},z)\exp_{\mathrm{D}}(u\bar{z})f(z) = f(u)$$

$$\int d\mu(\bar{z},z)\exp_{\mathrm{D}}(u\bar{z})\exp_{\mathrm{D}}(z\bar{w}) = \exp_{\mathrm{D}}(u\bar{w})$$

$$\int d\mu(\bar{z},z)\exp_{\mathrm{D}}(z\bar{z}t) = \sum_{n=0}^{p-1} t^n.$$

If A is an operator defined by equations (38)–(39), then

$$\int d\mu(\bar{z},z)A(z,\bar{z}) = \mathrm{Tr}(A) = \sum_{n=0}^{p-1} A_{n,n}.$$

These formulae indicate that the measure $d\mu(\bar{z},z)$ has the basic properties of a Gaussian measure.

Using this formulation the normal form of an operator can be introduced (see Itzykson and Zuber (1980)) and it is also interesting to study the deformed path integrals as given by Baulieu and Floratos (1991) without using the notion of a quantum plane. These topics are under investigation.

In this section we have shown that a well defined polynomial basis exists for the deformed oscillator with the same energy spectrum as the Pöschl–Teller potential. The extrapolation of these results to the ordinary q-deformed oscillator is straightforward: one simply replaces the function $F(x)$ in equation (19) by the corresponding function of the q-deformed oscillator

$$F(x) = \frac{\sin(\tau x/2)}{\sin(\tau/2)} \qquad \tau = 2\pi/p, p \text{ natural number.}$$

In this study we have not used the notion of a quantum plane; in the case of the q-deformed oscillator the quantum plane is used as an intermediate space where the quantum measure $\mathrm{d}\mu(\bar{z}, z)$ is calculated explicitly. In this paper the measure $\mathrm{d}\mu(\bar{z}, z)$ is defined through its fundamental property (36).

The interesting problem, which consistently arises, is clarification of the connection between the Hilbert space spanned by the eigenvectors $|n\rangle$ and the eigenfunctions of the Schrödinger equation with a Pöschl–Teller potential. The corresponding problem for the ordinary harmonic oscillator is the well known correspondence between the eigenstates $|n\rangle$ in the creation–destruction formalism and the Hermite polynomials weighted by a Gaussian measure, which are the eigenfunctions of the Schrödinger equation.

5. Results

In this paper we have constructed an algebra of operators:

$$\{a, a^\dagger, N, 1\}$$

satisfying the anticommutation relations:

$$[a, N] = a \qquad \text{and} \qquad [a^\dagger, N] = -a^\dagger \qquad [a, a^\dagger] = F(N+1) - F(N).$$

The function $F(x)$ is given by equation (19) and then this algebra corresponds to the energy spectrum of the Pöschl–Teller potential. For special values of the potential depth D there is a finite-dimensional irreducible representation of this algebra. The polynomial basis and the associated deformed integration and derivation are constructed. The method as it has been presented in this paper can be generalized for other energy spectra, while the case of the Coulomb energy spectrum is under investigation.

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